Modeling with Roots   Name: ___________________________ Period: ______

One of the challenges in landing people on the Moon in 1969 was escaping the Earth's gravitational pull. Fortunately, we have a nice equation to model that process created by Sir Isaac Newton:

$$V_{esc} = \sqrt{\frac{2GM}{R}}$$

where G is the gravitational constant, M is the mass of the planet, and R is the radius of the planet.

Note: Technically, the mass of the Earth is $5.98 \times 10^{24} kg$, but we scaled *everything* down to make life a little easier! We do not need to use this value and can use easier numbers in our domain.

Given that the radius of the Earth is 6370 km, and the gravitational constant G is $4932.4\ N(km/kg)^2$, we can model the different escape velocities needed for the Apollo 11 Moon Landing.

We know that the Earth is approximately 81 times as heavy as the Moon, so we can compare the two launches (one from the Earth to the Moon, and the return trip several days later) by thinking of the mass of the Moon as 1, and the mass of the Earth as 81 (on the x axis).

1. Graph the escape velocity equation $V(M) = \sqrt{\frac{2GM}{R}}$ with Escape Velocity (in km/second) on the y axis and Mass (in kg) on the x axis over the domain {x|0 ≤ x ≤ 100}. Use graph paper for this activity.
    a. Make sure you include the points $(1, V(1))$ and $(81, V(81))$ in your table for your graph.
2. Explain, in context, what this graph is telling you about the velocity required to escape gravity's pull in relation to the mass of the planet. Convert the velocity of $V(1)$ and $V(81)$ in terms of miles per hour. Is it fast?
3. Write the equation of a line tangent to the Escape Velocity curve at the points $(1, V(1))$ and $(81, V(81))$ Hint: find a point close to $(1, V(1))$ and $(81, V(81))$ and find the slope and equation of that line.
4. Compare the slopes of the two tangent lines. What do they tell you? You should interpret the slope of the line in the context of the problem. Are your findings surprising? Why or why not?
5. The area under the curve gives us the total amount of thrust necessary to escape the gravity well of the Earth and Moon. Find the total area under the curve for the Moon and the Earth launches. Hint: draw in trapezoids of equal base and find the area. You may need several trapezoids to find the area for the Earth launch.
    a. What does this number tell you about the amount of thrust needed for Earth versus the Moon?
    b. Why is there such a difference?
    c. Compare the amount of thrust needed to the velocity needed; what do you notice?
    d. If you used a trapezoid with a smaller base, would it give you a different area under the curve? What about larger bases? Which is the most accurate?

http://goo.gl/IiiUrh - Video of the lift off from moon of the lunar module

http://goo.gl/D3Rzb4 - Video of a space shuttle launch

$$y = \frac{1.21293 - 1.27517}{.95 - 1.05}(x - 1) + 1.2444 \approx .6224(x - 1) + 1.2444 \; is\ tangent\ at\ x = 1$$

$$y = \frac{11.19651 - 11.20343}{80.95 - 81.05}(x - 1) + 11.19997 \approx .0692(x - 1) + 11.19997 \; is\ tangent\ at\ x = 1$$

http://goo.gl/3ylCYA desmos file to model the escape velocity and demo in class.